Panel on Confidence Intervals for Likelihood Ratios

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Question 1 – A confidence interval for what?

- Bayes Theorem and the Likelihood Ratio
- E = evidence (E_x = crime scene; E_y = suspect)
- Hp = proposition that two samples have the same source
  Hd = proposition that two samples have a different source

$$\frac{\Pr(Hp \mid E)}{\Pr(Hd \mid E)} = \frac{\Pr(E \mid Hp)}{\Pr(E \mid Hd)} \frac{\Pr(Hp)}{\Pr(Hd)}$$

- Three terms:
  - Far right: "a priori" (before evidence) odds in favor of the common source proposition
  - Middle term: the likelihood ratio / Bayes factor
  - Far left term: "a posteriori" (after evidence) odds

Question 1 – A confidence interval for what?

$$BF = Pr(E \mid Hp) / Pr(E \mid Hd)$$

- Key issue is that probabilities often depend on parameters (e.g., copper example)

  $Pr(E \mid H, \theta)$

- Bayes Factor – Average over uncertainty in these parameters

  $Pr(E \mid H) = \int Pr(E \mid \theta, H)\, Pr(\theta \mid H)\, d\theta$

- The fully subjective Bayesian approach thus does not admit interval estimates (e.g., Taroni et al., LPR, 2016)

Question 1 – A confidence interval for what?

$$LR = Pr(E \mid Hp) / Pr(E \mid Hd)$$

- Key issue is that probabilities often depend on parameters (e.g., copper example)

  $$Pr(E \mid H, \theta)$$

- Likelihood ratio $LR = LR(\theta_p, \theta_d)$
  (the two parameter vectors may have elements in common)
- One **<u>can</u>** build a CI for this function – this would address sampling variability for parameter estimates

Question 2 – How would we use a CI?

- Evidence that decision makers have a very hard time using LRs
  - They often do not appear to understand the definition or interpretation
  - They do not appear to consistently update prior probabilities in a manner consistent with the LR interpretation
- If they could understand and interpret LRs, then it seems reasonable to assume they could probably handle CIs for LRs.
  - Indeed they might even desire it (conveys info on uncertainty)
- But …. that's a very big "if"!!

Question 3 – Can a CI approach do harm?

- The debate between dogmatic Bayesian statisticians and non-Bayesian statisticians can definitely do harm
  - To whom should the forensic community listen?
  - Will they tune the statistical community out?
  - Some indications of this in OSAC discussions
- A relevant anecdote:
  the ASA's recent p-value statement
- Note: OSAC Statistics Task Group has worked very well together across this divide

A role for "multiple" LRs

- Confidence intervals address sampling variability in parameter estimates
- More important to consider a range of LRs that address other factors (i.e., a sensitivity analysis)
  - Different parametric assumptions
  - Parametric vs non-parametric models (see, e.g., Lucy and Aitken, 2004)
  - Different estimation approaches for unknown parameters (diff't estimators, diff't databases)