CCQM 9: Redefinition of the kilogram

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Topics

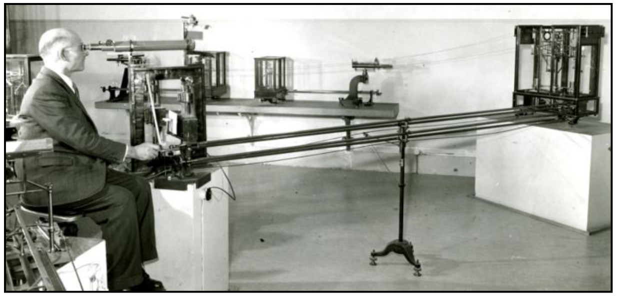
- Mass measurements on analytical balances.
- What's wrong with the present definition of the kilogram?
- Proposed 2018 redefinition of the kilogram in terms of the Planck constant (closely related to atomic mass).
- How will the redefinition be implemented?
- Are there consequences to users?

Present definition of the kilogram

"The <u>kilogram is the unit of mass</u>;

it is equal to the mass of the international prototype of the kilogram."

International prototype of the kilogram (IPK) put into service in 1889



Mass comparators ***have*** changed since 1889

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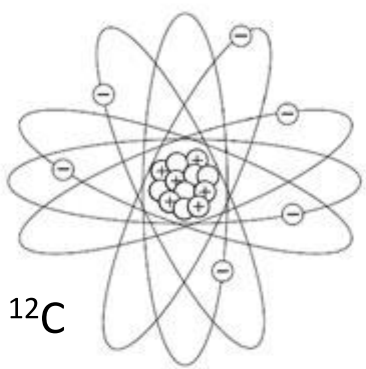
What the definition means

The mass in kilograms of any object X is given by :

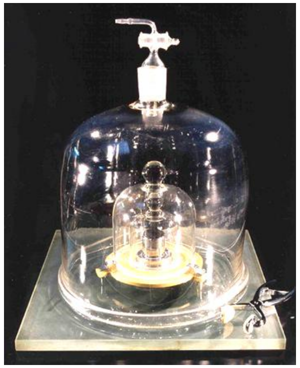
This ratio represents a <u>measurement</u> having an <u>experimental uncertainty</u>



$\sim 2 \times 10^{-26}$ kg

~ 0.1 kg

$$m_X = \left\{ \frac{m_X}{m_{IPK}} \right\} \text{kg}$$



$$m_X = \left\{ \frac{m_X}{\cancel{m_n}} \right\} \cdot \left\{ \frac{\cancel{m_n}}{\cancel{m_{n-1}}} \right\} \cdot \cancel{\ldots} \cdot \left\{ \frac{\cancel{m_2}}{\cancel{m_1}} \right\} \cdot \left\{ \frac{\cancel{m_1}}{m_{IPK}} \right\} \text{kg}$$

Traceability to the mass of the IPK, $m(\mathrm{IPK})$

The mass values of **Check Weights** are traceable to the mass of the IPK

The present SI works fine for analytical balances.

The new SI will also work fine for analytical balances.

The transition will be invisible.

From operator's manual:

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What atomic/physical constants are available to redefine the kilogram?

- Atomic masses:

  $m_a(\mathrm{X})$, where X is a nuclide such as $\mathbf{^{12}C}$, $^{28}\mathrm{Si}$, etc.

- Subatomic masses:

  electron mass $\boldsymbol{m_e}$ or proton mass m_p

- Physical constants:

  Planck constant $\boldsymbol{h}$ (SI unit: kg m^2 s^{-1} ; $E = h f$)

  atomic mass constant $\boldsymbol{m_u}$ ($= m_a(^{12}\mathrm{C})/12$)

  Newtonian constant G (SI unit: kg^{-1} m^3 s^{-2}), etc.

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Focus on atomic mass constant and Planck constant

- How to make high-accuracy experimental links?

$$m_{\mathrm{u}} = \left\{ \frac{m_{\mathrm{u}}}{m_{\mathrm{IPK}}} \right\} \mathrm{kg} \longrightarrow h = Q_{\mathrm{u}} \cdot m_{\mathrm{u}}$$

$$m = \left\{ \frac{m}{m_{\mathrm{IPK}}} \right\} \mathrm{kg} \longrightarrow h = Q \cdot m$$

Q_{u}, Q : combinations of (high-accuracy) auxiliary measurements.

h = 6.626…× 10^{-34} kg m^2/s ; SI unit of Q_{u}, Q is m^2/s

definitions of metre and second are not revised in the 'new' SI.

Two complementary experiments to consider

- **Watt balance method**

An electromagnetic balance weighs a mass $m \sim 1$ kg in terms of h and Q; quantum electrical devices are used.

Q combines **auxiliary measurements of two frequencies**, **one velocity**, the **local gravitational acceleration**, and dimensionless scaling factors. ***m* is traceable to $\boldsymbol{m}_{\mathbf{IPK}}$**

- X-ray crystal density (**XRCD**), also known as '**Avogadro**', **method**

Determines the atomic mass of ^{28}Si, $m_a(^{28}\mathrm{Si})$, and m_u by weighing a perfect crystal of mass $m \sim 1$ kg and determining the number of atoms in it. ***m* is traceable to $\boldsymbol{m}_{\mathbf{IPK}}$**

h is linked to $Q_u \cdot m_u$, where Q_u is <u>already</u> known to 0.45 parts in 10^9

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Simple example of XCRD method applied to ^{27}Al (published on web in *J. Chem. Educ.* earlier this month)

What Is a Kilogram in the Revised International System of Units (SI)?

Richard S. Davis, BIPM, 92312 Sèvres, France
J. Chem. Educ., Article ASAP
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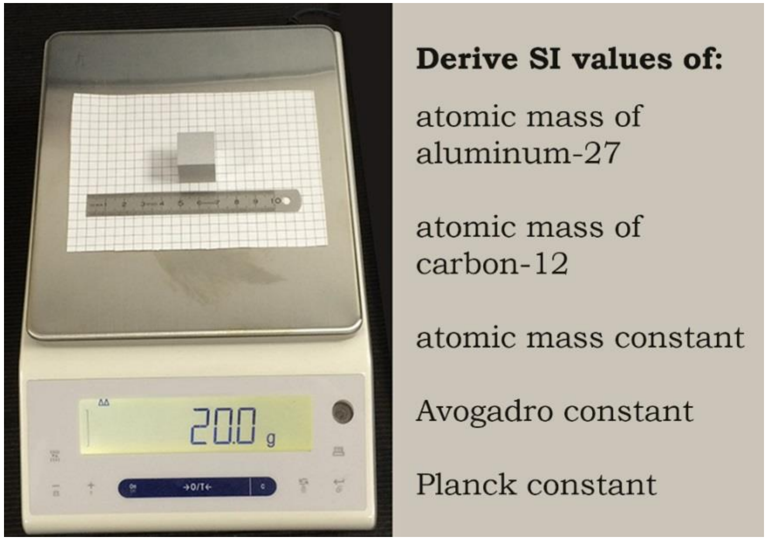

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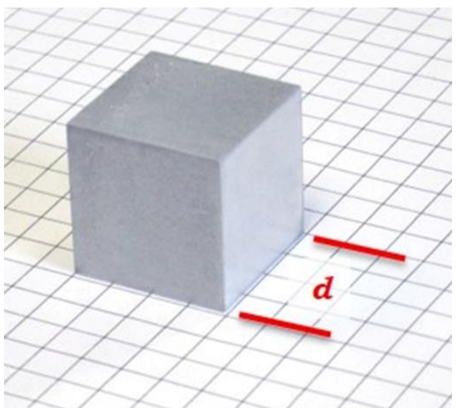
atomic mass of $^{27}\mathrm{Al}$, $m_a(^{27}\mathrm{Al})$ traceable to the mass of the IPK by the X-ray crystal density method (**XRCD**).

Make a 20-g cube of high-purity aluminum, with side *d* and mass *m*.

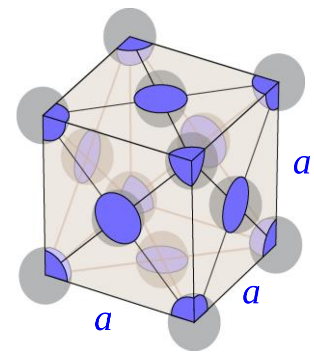
measured: *d* = 19.54 mm; *m* = 20.05 g.

If there are N atoms in the cube,



$$m = N \times m_a\left(^{27}\mathrm{Al}\right)$$

$$N = 4\frac{d^3}{a^3} = 4\left(\frac{d}{a}\right)^3$$



Al unit cell has 4 atoms.
a = 405 x 10^{-12} m

$$m_a\left(^{27}\mathrm{Al}\right) = \frac{m}{N} = \frac{m}{4}\left(\frac{a}{d}\right)^3$$

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$m_a(^{12}C)$ and the atomic mass constant, m_u

Relative atomic masses (atomic weights) are pure numbers.

The relative atomic mass of aluminum, which has only one natural isotope, is

$$A_r\left(^{27}\mathrm{Al}\right) = 12\frac{m_a\left(^{27}\mathrm{Al}\right)}{m_a\left(^{12}\mathrm{C}\right)} \longrightarrow m_a\left(^{12}\mathrm{C}\right) = \frac{12}{4}\frac{m}{A_r\left(^{27}\mathrm{Al}\right)}\left(\frac{a}{d}\right)^3$$

$$\frac{m_a\left(^{12}\mathrm{C}\right)}{12} = m_u = \mathrm{Da}$$

Note: the Avogadro constant N_A has now been measured to the same uncertainty as m_u and $m_a(^{12}C)$.

N_A = (1 g/mol)/m_u in the present SI.

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The Planck constant h from XRCD

The Bohr model of the hydrogen atom relates the Planck constant, h, to the the electron mass, m_e. As written today:

$$h \cdot (cR_\infty) = \frac{1}{2} m_e \cdot (\alpha c)^2$$

The Planck constant h from XRCD

The Bohr model of the hydrogen atom relates the Planck constant, h, to the the electron mass, m_e. As written today:

$$h = \left[\frac{\alpha^2 c}{2R_\infty}\right] m_e \longrightarrow h = \left[\frac{\alpha^2 c}{2R_\infty}\right] A_r(e)\, m_u = Q_u m_u$$

α : fine structure constant (a pure number)

c : speed of light in vacuum (in m/s)

$A_r(e)$: relative atomic mass of the electron (a pure number)

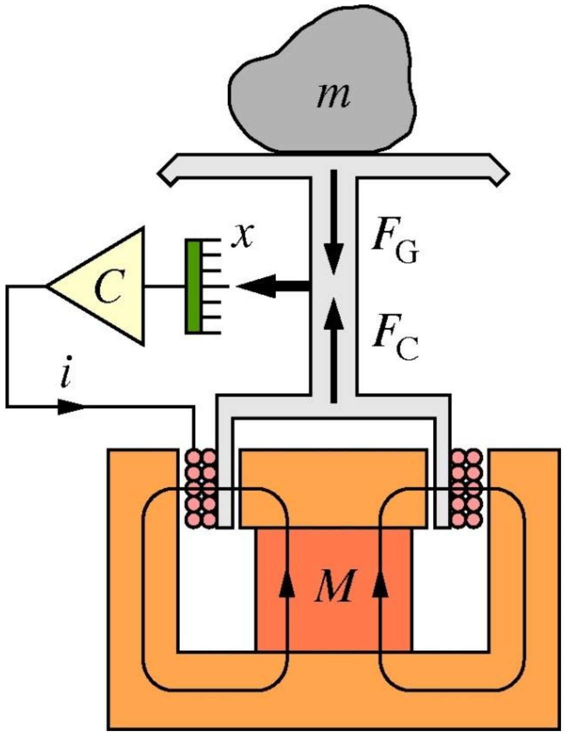
R_∞ : Rydberg constant (in 1/m)

relative uncertainty of Q_u is 0.45 parts in 10^9 (CODATA 2014)

$$h = \left[\frac{Q_u}{A_r\left({}^{27}\mathrm{Al}\right)} \frac{a^3}{4d^3}\right] m$$

$$h = \left[\frac{Q_u}{A_r\left({}^{28}\mathrm{Si}\right)} \frac{a_{Si}^3}{8\left(\pi/6\right) d^3}\right] m = Q_{SiXRCD} \cdot m$$

The Planck constant *h* from a watt balance – Step 1



from Mettler-Toledo documentation describing servocontrol of an analytical balance.

Gravitational force

$$F_G = mg$$

Electro-magnetic force

$$F_C = i\,(BL)$$

$$mg = i\left(BL\right) = \frac{V'}{R}\left(BL\right)$$

An analytical balance is calibrated by an internal or external mass standard traceable to $m(\mathrm{IPK})$

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watt balance – Step 2

- Move the coil of wire (length L) vertically through the magnetic field (B) at a controlled velocity (v).

- A voltage difference V appears across the ends of the wire:

$$V = v(BL)$$

$$mg = \frac{V'}{R}(BL)$$

(from previous slide)

- Combining equations, **mechanical power = electrical power**

$$mgv = V\frac{V'}{R}$$

watt balance – the link to the Planck constant

- Measure V, V' and R with **quantum electrical devices**, first discovered in the last century and now widely used:

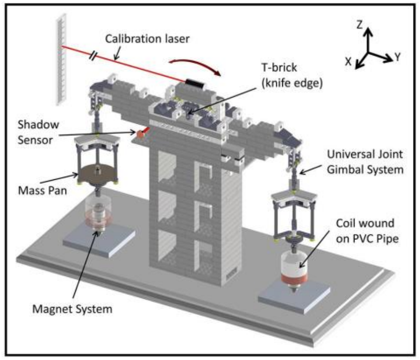
$$mgv = V\frac{V'}{R}$$

$$h = \left[4b\frac{gv}{f \cdot f'}\right]m = Q_{\mathrm{wb}} \cdot m$$

f and f' are **microwave frequencies** associated with the voltage measurements;

b is a product of integers and dimensionless scaling factors;

$$Q_{\mathrm{wb}} = \left[4b\frac{gv}{f' \cdot f}\right]$$



facebook.com/LEGOwattbalance

Redefining the kilogram in terms of h and realizing the new definition: The XCRD route

Determine the value of h in the present SI, using all possible high_accuracy methods, **for example by Si XRCD** :

$$m_{\mathrm{IPK}} \rightarrow m \rightarrow m_{\mathrm{u}} \rightarrow h/Q_{\mathrm{u}} \rightarrow \boldsymbol{h}_{\mathbf{exp}}$$

$u_r = 20 \times 10^{-9}$ (from m_{IPK} to m_{u}); $u_r < 1 \times 10^{-9}$ (from m_{u} to h_{exp})

Redefine the kilogram by making the value of h exactly equal to $\langle h_{\mathrm{exp}} \rangle$

$$\langle h_{\mathrm{exp}} \rangle \rightarrow h \text{ (exact)}$$

'Realize' the new definition, for example by the Si XRCD method

$$h \rightarrow h/Q_{\mathrm{u}} \rightarrow m_{\mathrm{u}} \rightarrow m \rightarrow m_y$$

$u_r < 1 \times 10^{-9}$ (from h to m_{u}); $u_r = 20 \times 10^{-9}$ (from m_{u} to m_y)

m_y is the mass of any suitable macroscopic object y, <u>such as the IPK</u>

Redefining the kilogram in terms of *h* and realizing the new definition: The watt balance route

Determine the value of h in the present SI, using all possible high_accuracy methods, **for example with a watt balance**:

$$m_{\mathrm{IPK}} \rightarrow m \rightarrow h/Q_{\mathrm{wb}} \rightarrow \boldsymbol{h}_{\mathbf{exp}} \rightarrow h_{\mathrm{exp}}/Q_{\mathrm{u}} \rightarrow m_{\mathrm{u}}$$

$|\leftarrow u_r = 20 \times 10^{-9} \rightarrow|\leftarrow u_r < 1 \times 10^{-9} \rightarrow|$

Redefine the kilogram by making the value of h exactly equal to $\langle h_{\mathrm{exp}} \rangle$

$$\langle h_{\mathrm{exp}} \rangle \rightarrow h \text{ (exact)}$$

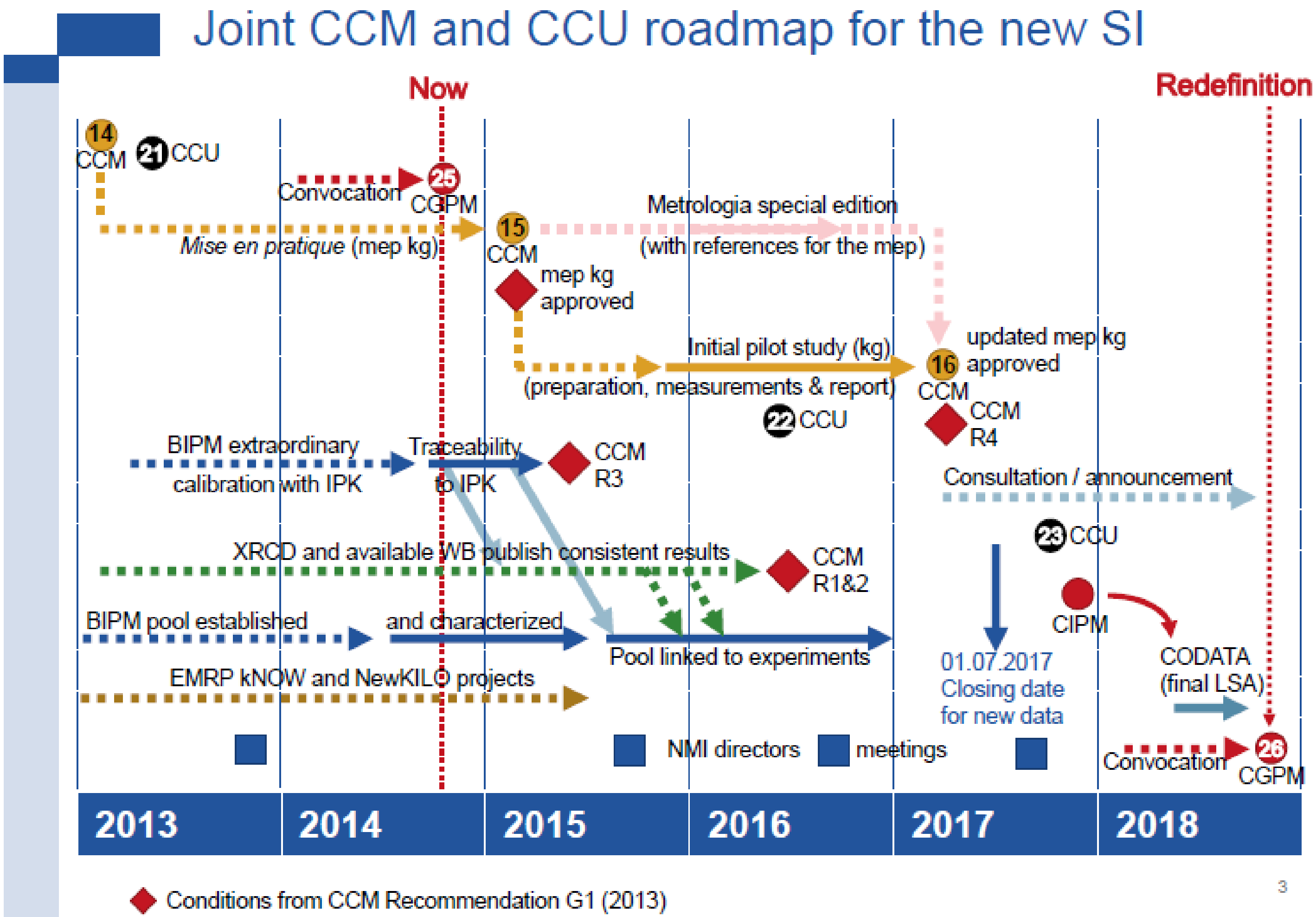
'Realize' the new definition, for example by the watt balance method

$$h \rightarrow h/Q_{\mathrm{wb}} \rightarrow m \rightarrow m_y$$

$|\leftarrow u_r = 20 \times 10^{-9} \rightarrow|$

m_y is the mass of any suitable macroscopic object y, such as the IPK

The CCM – CCU Roadmap (still on schedule)



Impact on users

Short to medium term

- On-demand availability of primary calibrations from (an increasing number of) NMIs and the BIPM.
- Fixed-valued Planck constant brings quantum-based electrical metrology fully into the SI.
- kilogram unit will be inherently stable; problems due to an unstable mass unit eliminated 'in principle' (so they will never be seen in practice).
- Vast majority of users will not notice the difference.

Long term

- Primary realizations may decrease in cost and extend to wider ranges of application.

Thanks for your attention